Poisson Regression Model with Application to Doctor Visits

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ABSTRACT

The problem was to apply Poisson regression and its variants to modeling doctor visits. The set of explanatory variables under consideration was tested and subsequently the final model was determined. The MLE. Was used was for estimation using the Stata software package. The research resulted in selecting the count model variant with the best estimates using information criteria. In addition, where there was overdispersion, Negative binomial regression gave better estimates then Poisson model.

KEYWORDS: *Poisson Regression , Model, Application*

INTRODUCTION

1.1 BACKGROUND OF STUDY

A visit to doctor, also known as physician office visit or ward round is a meeting between a patient with a physician to get health advice of treatment for a symptom or condition. According to a survey in the in the United States a physician typically sees between fifty and one hundred patients per week, but the rate of visitation may vary with medical specialty, but differs one little by community size. The four great cornerstone of diagnostic medicine are anatomy (structure what is there), physiology (how the structure work) pathology (what goes wrong with anatomy and physiology) and psychology (mind and behavior), the physician should consider the patient in their "well" context rather than simply as a walking medical condition. This means the socio-political context of the patient (family, work, stress, beliefs) should be assessed as it often offers vital clues to the patient's condition and further management. The attempt to solve physical problems led gradually to a statistical data type. In the analysis of data it is necessary to first comprehend the type of data before deciding the modeling approach to be used in the context of modeling the discrete, non negative nature count of a dependent variable, the use of least square regression models several methodological limitations and statistical properties (Miaou, 1993; Karlaftis and tarko,1998; Shankar,1995). Unlike the classical linear regression models for count and counts are non-negative integer (0,1,2,3…) and these integers arise from counting rather than ranking. Count when related to other variables would be treated as dependent variables. The Poisson regression model is a good starting point of count data modeling. Many examples such as visits to doctor, the number of patent awarded to a firm, the number of road accident death, the number of dengue fever cases are restricted to a single digit or integer with quite low number of events (Cameron and

Trivedi, 1998, Hausman et al, 1984; Radin et al, 1996). For such feature of data, Poisson regression has more advantages over conventional linear model (Chin and Quddus, 2003; Shankar, 1995). Poisson regression suffers one potential problem, this is related to the assumption of equality of the mean and variance a property called equidispersion. When this assumption is violated, for instance the variances excess the mean, an overdispersion occurs. Failure to control for overdispersion will lead to inconsistent estimates, biased in standard error and inflated test statistics. Hence in modeling count data, it is a usual practice after the development of Poisson regression model to proceed with analysis of correcting for overdispersion if it exists. One of the approaches to modeling overdispersion is to use quasi likelihood estimation techniques proposed by Wedderbum (1974). The analysis of data in this study focuses on the use of Poisson regression with application to visits to Doctor. This is because Poisson regression has more advantages over conventional linear model.

Therefore, it appears worthwhile to devote effort in using Poisson regression to modeling doctor's home visit among the aged in Nigeria with a view of evaluating the impact of doctors home visit among the aged.

METHODOLOGY

Having reviewed related work on count data model-Poisson regression, it is of need to gather data which will be utilized as a part of figuring a reasonable and practical model for this project. Conversely, this chapter depicts the methodology and its point of utilization. It also clarifies the research method and the research that should be utilized and the techniques utilized to guarantee the unwavering quality and legitimacy of the research.

3.1 Count Data Model

Count data is a statistical data type in which the observations can take only non- negative integer values {0,1,2, 3,...} and integers arise from counting. An individual piece of count data is often termed a count variable that is count variable indicates the number of times something happened. When count variable is treated as a random variable, the Poisson distribution is commonly used to represent its distribution. We have several count models bur in the project we consider Poisson regression model (PRM), which is one of the foundation of other count models.

3.2 Poisson Distribution

In Probability Theory and Statistics, the Poisson distribution is a discrete Probability distribution that expresses the Probability of a given number of events occurring in a fixed interval of time or space. If these occurs with a known constant rate and independently of the time since the last event. The Poisson distribution can also be used for the number of events in other specified intervals such as distance, area or volume. The Poisson distribution is popular for modeling the number of time an event occurs in an interval of time or space. Example are The Poisson distribution may be useful to model events such as: The number of Meteorites greater than 1 meter diameter that strike Earth in a year, The number of Patients arriving in an Emergency room between 10pm to 11pm, The number of Photons hitting in particular time interval and the number of Doctor's visit to Patients per annual.

3.2.2 ASSUMPTIONS AND VALIDITY

The Poisson distribution is an appropriate model if the following assumptions are true where kIs the number of times an event occurs in an interval and k can take values 0,1,2,….The occurrences of one event do not affect the Probability that a second event will occur. That is, events occur independently, The average rate at which events occurs is constant, Two events cannot occur at exactly the same instant instead at each very small sub-interval exactly one event either occurs or does not occur, The actual Probability distribution is given by a binomial distribution and the number of trials is sufficiently bigger that the number of successes one is asking about and If these conditions are true then is a Poisson random variable and the distribution of k is a Poisson distribution.

3.2.3 Probability of Events For A Poisson Distribution

An event can occur 0,1,2,3,… times in an interval. The average number of events in an interval is designatedλ(lambda). λ is the event rate, also called the rate parameter. The Probability of observing K event in an interval is given by the equation

$$P(keventsininterval) = \frac{e^{-\lambda}\lambda^{k}}{K!}$$

Where , λ is the average number of events per interval. e is the number 2.71828 …. (Euler's number) the base of the natural logarithms. K takes the values$0,1,2,3$ … ..

$K!$ =$k \times (k-1) \times (k-2) \times \ldots \times 2 \times 1$ is the factorial of K.

This equation is the Probability mass function (PMF) for a Poisson distribution. The equation can be adapted if instead of the average number of events λ, we are given a time rate r for the events to happen. Then λ$= rt$ (with r in unit $1/$time), and$P(Keventsinintervalt) = e^{-rt}\frac{(rt)k}{k!}$

3.2.4 DEFINITION

A discrete random variable x is said to have a Poisson distribution with parameter λ> 0, if for $k = 0,1,2, \ldots$, the Probability mass function of x is given by

$$F(k;\lambda) = \Pr(X = x) = \frac{\lambda^{k}e^{-\lambda}}{k!},$$

The positive real number λ is equal to the expected value of x and also to us variance.

$$\lambda = E(x) = Var(x).$$

The Poisson distribution can be applied to systems with large number of possible events, each of which is rare. How many such events will occur during a fixed time interval? Under the right circumstances, this is a random number with a Poisson distribution. The conventional definition of Poisson distribution contains two terms that can easily overflow on Computers λ^k to $k!$ can also produce a rounding error that is very large compared to $e^{-\lambda}$ and therefore give an erroneous result. For numerical stability the Poisson Probability Mass Function should therefore be evaluated as : $F(k;\lambda) = \exp\{\mathrm{k}\ln\lambda - \lambda - \ln\Gamma(\mathrm{k}+1)\}$ which is Mathematically equivalent but numerically stable. The natural logarithm of the Gamma Function can be obtained.

3.3 POISSON REGRESSION

Poisson regression is a generalized linear model (GLM is a flexible generalization of ordinary linear regression that allows response variable that have error distribution models other than normal distribution) form of regression analysis used to model count data and contingency tables. Poisson regression assumes the response variable Y has a Poisson distribution and assumes the logarithm of its expected value can be modeled by a linear combination of unknown parameters. A Poisson regression model is sometimes known as a log-linear model, especially when used to model contingency tables.

3.3.1 POISSON REGRESSION MODEL

Is $x \in \mathbb{R}^n$ is a vector of independent variable, then the model takes the form

$\log\left(E\left(\frac{y}{x}\right)\right) = \alpha + \beta' x$ where $\alpha \in \mathbb{R}$ and $\beta \in \mathbb{R}^n$. Sometimes this is written more compactly as $\log\left(E\left(\frac{y}{x}\right)\right) = \theta' x$, where x is now an (n +1)- dimensional vector consisting of n independent variables concatenated to a vector of one. Here θ is simply x concatenated to β. Thus, when given a Poisson regression model θ and an input vector x, the predicted mean of the associated Poisson distribution is given by

$$E\left(\frac{y}{x}\right) = \theta'^{x}.$$

3.3.2 Poisson Regression Model Variables

In Poisson regression response/outcome variable Y is a count. But we can also have $\frac{y}{t}$,the rate (or incidence) as the response variable, where t is an interval representing time, space or some other grouping. Explanatory variable(s),$X = (x_1, x_2, \ldots\ldots x_k)$ can be continuous or a combination of continuous and categorical variables. Convention is to call such a model "Poisson regression".

If y/t is the variable of interest then even with all categorical predictors, the regression model will be known as Poisson regression model and If Y_i are independent observations with corresponding values x_i of the predictor variables, then θ can be estimated by Maximum Likelihood. The Maximum-likelihood estimates lack a closed-form expression and must be found by numerical methods. The Probability surface for Maximum-likelihood Poisson regression is always concave, making Newton-Raphson or other gradient based methods appropriate estimation techniques.

3.3.3 MAXIMUM LIKELIHOOD BASED PARAMETER ESTIMATES.

Given a set of parameter θ and an input vector$x$, the mean of the predicted Poisson distribution, as stated earlier is given by $P\left(\frac{y}{x};\theta\right) = (\lambda^{y}/y!)e^{-\lambda} = e^{y\theta\prime x}e^{e^{-\theta\prime x}}/y!$.

Now suppose we are given a data set consisting of m vectors$x_i \epsilon \mathbb{R}^{n+1}$, i=1,2,…, m along with a set of m values $y_1, \ldots\ldots, y_m \in \mathbb{N}$. Then, for a given set of parameterθ, the Probability of attaining this particular set of data is given by

$$P(y_1, \ldots., y_m/x_1, \ldots., x_m; \theta) = \prod_{i=1}^{m} e^{y_{i\theta'x_i}} e^{-e^{\theta' x_i}} / y_{i!}$$

By the method of Maximum likelihood, we wish to find the set parameters θ that makes this Probability as large as possible. To do this, the equation is first rewritten as a likelihood function in terms of θ:

$$L\left(\frac{\theta}{X,Y}\right) = \prod_{i=1}^{m} e^{y_{i\theta'x_i}} e^{-e^{\theta' x_i}} / y_{i!}$$

Note that the expression on the right hand side has not actually changed. A formula in this form is typically difficult to work with; instead one uses the$log - likelihood$.

$$\ell\left(\frac{\theta}{X,Y}\right) = \sum_{i=1}^{m} (y_i \theta^i e^{\theta^i x_i}) \; .$$

Notice that the parameter θonly appears in the first two terms of each term in the summation. Therefore, given that we are only interested in finding the best value of θwe may drop the y_i! and simply write

$$\ell(\theta/X,Y) = \textstyle\sum_{i=1}^{m} y_i \theta' x_i - e^{\theta' x_i}.$$

To find a Maximum likelihood, we need to solve an equation

$$\ell \frac{(\theta/X,Y\,)}{\delta\theta} = 0$$

This has no closed-form solution. However, the negative, log-likelihood $-\ell(\theta/X,Y)$ is a convex function and so standard convex optimization techniques such as gradient descent can be applied to find the optimal value ofθ.

3.3.4 Poission Regression in Practice

Poisson regression may be appropriate when the dependent variable is a count, for instance the event of doctor's visit. The event must be independent in the sense that the arrival of one Patient will not make another less or more likely but the Probability per unit time of events is understood to be related to covariates such as time of day.

3.3.5 Exposure and Offset

Poisson regression may also be appropriate for the rate of data, where the rate is a count of events divided by some measure of the unit's exposure (a particular unit of observation). For example, biologist may count the number of tree observations, exposure would be unit area, and rate would be the number species per unit area. In Poisson regression that is handled as an offset, where the exposure variable enters on the right-hand side of the equation but with parameter estimation (for log (exposure)) constrained to 1.

$\log\left(\left(E(Y/x)\right) = \log(exposure) + \theta^{i}x\right.$which implies

$$\log\left(E(Y/x)\right) = \log(exposure) = \log\left(\frac{E\,Y/x}{exposure}\right) = \theta' x.$$

3.3.6 OVERDISPERSION

A characteristic of Poisson distribution is that its mean is equal to its variances. In certain circumstances, it will be found that the observed variances that the observed variances is the greater than the mean, this is known as overdispersion and indicates the model to be appropriate. A common reason is the omission of relevant explanatory variable or dependent observations. Under some circumstances, the problem of overdispersion can be solved by using quasi-likelihood estimation or a negative binomial distribution instead. Ver Hoef and Boveng described the difference between quasi-Poisson (also called overdispersion with quasi-likelihood) and negative binomial (equivalent to gamma-Poisson) as follows If $E(Y) = \mu$The quasi-Poisson model assumes var(Y) =$\theta\mu$ while the gamma-Poisson assumes $var(Y) = \mu(1 + k\mu)$ is where θ the quasi-Poisson over dispersion parameter, and k is the shape parameter of the negative binomial distribution for both models, parameters are estimated using iteratively reweighted least squares. For quasi-Poisson, the weights areμ/θ.

3.4 NEGATIVE BINOMIAL REGRESSION MODEL

In negative binomial regression, the mean of y is determined by the exposure time t and a set of k regressor variables $the\ x's$. the expression relating these qualities is

$$\mu_i = \exp(\ln(t_i) + \beta_1 x_{1i} + \beta_2 x_{2i} + \cdots + \beta_k x_{ki})$$

Often, $x_1 \equiv 1$, in which case β_1 is called the intercept. The $\beta_1,\ \beta_2,\ \beta_3$ are unknown parameters that are estimated from a set of data, their estimates are symbolized as b_1, b_2, b_3.

The fundamental negative binomial regression model for an observation i is given as;

$$P_r\,(Y = y_i/\mu_i, \alpha) = \frac{\Gamma(y_i + \alpha^{-1})}{\Gamma(\alpha^{-1}\Gamma(y_i + 1))}\left(\frac{1}{1 + \alpha\mu_i}\right)^{\alpha^{-1}}\left(\frac{\alpha y_i}{1 + \alpha\mu_i}\right)^{y^i}$$

3.4.1 MAXIMUM LIKELIHOOD BASED ESTIMATION

The regression coefficients are estimated using the method of maximum likelihood. The logarithm of the likelihood function is given as
$\mathcal{L} = \sum_{i=1}^{n}\{\ln[\Gamma(y_i + \alpha^{-1})] - \ln[\Gamma(\alpha^{-1})] - \ln[\Gamma(y_i + 1)] - \alpha^{-1}\ln(1 + \alpha_i\mu_i) - y_i\ln(1 + \alpha\mu_i) + y_i\ln(\alpha) + y_i\ln(\mu_i)\}.$

$\mathcal{L} = \sum_{i=1}^{n}\{(\sum_{j=0}^{y_{i-1}}\ln(j + \alpha^{-1})) - \ln(\Gamma(y_i + 1)) - (y_i + \alpha^{-1})\ln(1 + \alpha_i\mu_i) + y_i\ln(\mu_i) + y_i\ln(\alpha)\}$ the first derivative of $\mathcal{L}$

$\frac{\delta\mathcal{L}}{\delta\beta_j} = \sum_{i=1}^{n}\frac{x_{ij}(y_i-\mu_i)}{1+\alpha\mu_i}$, j=1, 2, 3, ….,$k$

$$\frac{\delta\mathcal{L}}{\delta\alpha} = \sum_{i=1}^{n}\{\alpha^{-2}(\ln(1 + \alpha\mu_i) - \sum_{j=1}^{y_{i-1}}\frac{1}{j + \alpha^{-1}}) + \frac{y_i - \mu_i}{\alpha(1 + \alpha\mu_i)}\}.$$

$$-\frac{\delta^2\mathcal{L}}{\delta\beta_r\delta\beta_s} = \sum_{i=1}^{n}\frac{\mu_i(1 + \alpha\mu_i)x_{ir}x_{i5}}{(1 + \alpha\mu_i)^2}$$

$r, s = 1, 2, \ldots, k$

$$-\frac{\delta^2\mathcal{L}}{\delta\beta_r\delta\beta_s} = \sum_{i=i}^{n}\frac{\mu_i(y_i - \mu_i +)x_{ir}}{(1 + \alpha\mu_i)^2}$$

$r = 1, 2, \ldots., k$

$$\frac{\delta^2\mathcal{L}}{\delta\alpha^2} = \sum_{i=1}^{n}\{\sum_{j=0}^{y_{i-1}}\left(\frac{j}{1 + \alpha_j}\right)^2 + 2\alpha^{-3}\ln(1 + \alpha\mu_i) - \frac{2\alpha^{-2}\mu_i}{1 + \alpha\mu_i} - \frac{(y_i + \alpha^{-1})^2}{(1 + \alpha\mu_i)^2}\mu_i\}$$

Equating the gradients to zero gives the following set of likelihood equations

$$\sum_{i=1}^{n}\frac{x_{ij}(y_i - \mu)}{1 + \alpha\mu_i} = 0 \quad j = 1, 2, .., k$$

$$\sum_{i=1}^{n}\{\alpha^{-2}(\ln(1 + \alpha\mu_i) - \sum_{j=0}^{y_{i-1}}\frac{1}{j + \alpha^{-1}}) + \frac{y_i + \mu_i}{\alpha(1 + \alpha_i\mu_i)}\} = 0.$$

3.5 ZERO INFLATED MODELS

Zero inflated model is a statistical model based on a zero-inflated probability distribution, that is a distribution that allows for frequent zero-valued observations.

3.5.1 ZERO INFLATED POISSON (ZIP)

One well known zero-inflated model is Diane Lambert's zero inflated poisson model, which concerns a random event containing excess zero-count data in unit time.

The zero inflated poisson (ZIP) model employs two components that correspond to two zero generating processes. The first process is governed by a binary distribution that generates structural zeros. The second process is governed by a poisson distribution that generates counts some of which may be zero. The two model components are describes as follows

$$P_r(y_i = 0) = \pi + (1+\pi)e^{-\lambda}$$

$$P_r(y_i = h_i) = (1-\pi)\frac{\lambda h_i e^{-\lambda}}{h_i\,!}$$

Where the outcome variable y_i has any non-negative integer value, λ is the expected poisson count for i^{th} individual; π is the probability of extra zeros. The mean is $(1-\pi)\lambda$ and the variance is $\lambda(1-\pi)(1+\pi\lambda)$.

3.5.2 ESTIMATES OF ZERO INFLATED POISSON

The method of moments estimator are given by

$$\hat{\lambda}_{m_0} = \frac{s^2+m^2}{m} \;\; -1,$$

$$\widehat{\Pi}_{m_0} = \frac{s^2 - m}{s^2 + m^2 - m}$$

Where m is the sample mean and s^2 is the sample variance. The maximum likelihood estimator is derived from the following equation

$$\bar{x}\left(1 - e^{\hat{\lambda}_{ml}}\right) = \hat{\lambda}_{ml}\left(1 - \frac{n_0}{n}\right).$$

Where $\bar{x}$ is the sample mean, and $\frac{n_0}{n}$ is the observed proportion of zeros. This can be solved by iteration and the maximum likelihood estimator for π is given by

$$\widehat{\Pi}_{ml} = 1 - \frac{\bar{x}}{\hat{\lambda}_{ml}}$$

3.5.3 ZERO INFLATED NEGATIVE BINOMIAL (ZINB) REGRESSION MODEL

The zero-inflated negative binomial (ZINB) regression model is used for count data that exhibit overdispersion and excess zeros. The data distribution combines the negative binomial distribution and the log it distribution. The positive value of Y are the nonnegative integers:0, 1, 2, and so on. The probability distribution of the ZINB random variable y_i can be written as;

$$P_r(y_i = j) = \begin{cases} \pi_i + (1+\pi_i)g(y_i = 0) & if\ j = 0 \\ (1-\pi_i)g(y_i) & if\ j > 0 \end{cases}$$

Where $\pi_i$is the logistic link function defined below and $g(y_i)$ is the negative binomial distribution given by :$g(y_i) = P_r(Y = \mu_i, \alpha) = \frac{\Gamma(y_i+\alpha^{-1})}{\Gamma(\alpha^{-1})\Gamma(y_i+1)}\left(\frac{1}{1+\alpha\mu_i}\right)^{\alpha^{-1}} (\frac{\alpha\mu_i}{1+\alpha\mu_i})^{y_i}$

Exposure time t can be included in the negative binomial component and a set of k regressor variables ($the\ x's$) the expression relating the equation is

$$\mu_i = \exp(ln(t_i)) + \beta_1 x_{1i} + \beta_2 x_{2i} + \cdots. + \beta_k x_{ki}$$

Often, $x_1 \equiv 1$, the logistic link function π_i given by;

$$\pi_i = \frac{\lambda_i}{1+\lambda_i}\ where\ \lambda_i = \exp(\ln(t_i)) + Y_1 Z_{1i} + Y_2 Z_{2i} + \cdots. + Y_m Z_{im})$$

Where ($the\ z's$) are the regressor variables and t is the exposure time.

3.5.4 MAXIMUM LIKELIHOOD ESTIMATE FUNCTION

The regress or coefficients are estimated using the method of maximum likelihood. The logarithm of the function is

$$L = L_1 + L_2 + L_3 - L_4$$

$$where\ L_1 = \sum_{\{i:y_i=0\}} \left[\lambda_i + (1+\alpha\mu_i)^{-\alpha^{-1}}\right]$$

$$L_2 = \sum_{\{i:y_i>0\}} \sum_{j=0}^{y_i-1} ln(j + \alpha^{-1})$$

$$L_3 = \sum_{\{i:y_i>0\}} \{-\ln(y_{i!}) - (y_i + \alpha^{-1})\ln(1+\alpha\mu_i) + y_i \ln(\alpha) + y_i \ln(\mu_i)\}$$

$$L_4 = \sum_{i=1}^{n} \ln(1+\lambda_i).$$

3.6 AKAIKE INFORMATION CRITERION (AIC)

AIC is an estimator of the sample prediction error and thereby relative quality of statistical models for a given set of data. Given a collection of models for the data, AIC estimates the quality of each model, relative to each of the other models.

In estimating the amount of information lost by a model, AIC deals with the trade-off between the goodness of fit of the model and the simplicity of the model. AIC is commonly used to fit statistics, it has two formulations;

AIC(1) = $-2[l-k]$ and AIC(n) = $-\frac{2}{n}(l-k)$

where k is the number of predictors including the intercept.

3.7 BAYESIAN INFORMATION CRITERION (BIC)

BIC is a model selection among a finite set of models. The model with lower BIC is preferred. It is based, in part, on the likelihood function and it is closely related to AIC, it used to fit statistics.

It has three formulation;

BIC $(R) = D - (df)\ln(n)$

$BIC\ (L) = -2l + kln(n)$

$BIC\ (Q) = -n(l - kln(k))$

Where df is the residual degree of freedom.

3.8 CHI-SQUARE TEST STATISTICS

X^2 measures the distance between the observed and expected counts across all cells and is computed as $X^2 = \sum_{all\ cells} \frac{(O-E)^2}{E}$, Where O is the observed count x_j and E is the expected count $n\hat{\pi}_j$

3.8.1 P-VALUE

This is the significant level of the chi-square test. This is the probability that s chi-square value with degree of freedom DF is equal to the value or greater. If the value is less than 0.05 (or other appropriate value)the term is said to be statistically significant

RESULTS

Table 4.1 description of data

The table shows the basic information about the data file, it displays the number of observations in the file from the output we see that the number of observations is 4,412, the number of variables which is 10, the size of the file which is 75,004, the variable name which are docvis, age, income, female, black, Hispanic, married, physlim private and chronic, storage type, display format and variable label.

Table 4.2 Summary of data

The table provides information about the data file, which includes variable (docvis, married, female, physlim, private, chronic, income), number of observations (obs), mean (estimated values of the mean for each variable), standard deviation (std.Dev.), minimum (Min) and maximum(Max)

Table 4.1 description of data

```
obs:        4,412
vars:          10
```

Variable	Obs	Mean	Std. Dev.	Min	Max
docvis	4,412	3.957389	7.947601	0	134
married	4,412	.6357661	.4812692	0	1
female	4,412	.4718948	.4992661	0	1
physlim	4,412	.1656845	.3718393	0	1
private	4,412	.7853581	.4106202	0	1
chronic	4,412	.3263826	.4689423	0	1
income	4,412	34.34018	29.03987	-49.999	280.777

```
chronic         byte    %8.0g               = 1 if a chronic condition
```

Table 4.2 summary of data

Table 4.3 Poisson Regression

The output of the table begins with the iteration log, which gives the values of the log likelihood starting with the null model. The last value in the iteration log is the final value of the log likelihood for the full model and is displayed again. The header information is presented next, on the right-hand side, the number of observations used in the analysis (4,412) is given alone with the Wald chi-square statistic with 7 degree of freedom for the full model, followed by the P-value for the chi-square. The header also includes pseudo-R^2 below the header we have docvis which is the response variable in the poisson regression. Underneath docvis are the predictor variables and the intercept(_const.). The poisson regression coefficient, these are the estimated poisson regression coefficient for the model for each of the variables along with standard errors, z-score and p-values. The estat gof shows the pearson and deviance goodness of fit chi-square test.

Table 4.3 poisson regression

```
poisson docvis age income female married physlim private chronic
teration 0:   log likelihood = -18120.776
estat gof

        Deviance goodness-of-fit  =  36983.83
        Prob > chi2(4411)         =    0.0000     obs   =      4,412
                                                        =    9621.15
        Pearson goodness-of-fit   =   70404.5           =     0.0000
        Prob > chi2(4411)         =    0.0000           =     0.2098
```

docvis	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
age	.0313875	.0077222	4.06	0.000	.0162522	.0465228
income	.0040537	.0002443	16.60	0.000	.003575	.0045324
female	.4917597	.0161212	30.50	0.000	.4601628	.5233566
married	-.0180317	.0159971	-1.13	0.260		
physlim	.4634731	.0173834	26.66	0.000		
private	.7746199	.0278532	27.81	0.000		
chronic	.979682	.016489	59.41	0.000		
_cons	-.4030394	.0411802	-9.79	0.000		

Table 4.4 Negative Binomial Regression

The dispersion on the top of the table refers to how the over dispersi method is mean dispersion. The log likelihood is the log likelihood of tl of obs, is the number of observation used in the regression model. LR chi2(7), is the test statistics that all regression coefficients in the model are simultaneous equal to zero. Prob >chi2, is the probability of getting a LR test statistic as extreme as, or more so, than the observed under the null hypothesis. Pseudo R^2, this is the McFadden's R-square. Docvis, this is the response variable underneath are the predictor variables, the intercept and the dispersion parameter. Coef. are the estimated negative binomial coefficients for the model. Std.Err these are the standard errors for the regression coefficients and the dispersion parameter for the model. Z and P>|z|, these are the test statistics and p-value, respectively. /alpha, is the estimate of log of the dispersion parameter. Alpha this is the estimate of the dispersion parameter.

4.5 Zero Inflated Poisson Regression

The table begins with the iteration log giving the values of the log likelihoods starting with a model that has no predictors. Next comes the header information on the right-hand side the number of observations used, number of non zero observations are given alone with the likelihood ratio chi-squared this is followed by the p-value for the chi-square. Below the header we find our poisson coefficient for each count predicting variables along with Standard errors, z-score, p-value. Following these are the logit coefficients for the variable predicting excess zeroes along with its standard errors, z-scores, and p-values

Table 4.4 negative binomial regression

```
5 . nbreg docvis age income female married physlim private chronic , nolog

Negative binomial regression                    Number of obs     =      4,412
                                                LR chi2(7)        =    1213.39
Dispersion     = mean                           Prob > chi2       =     0.0000
Log likelihood = -9782.2197                     Pseudo R2         =     0.0584
```

docvis	Coef.	Std. Err.	z	P>|z|
age	.081139	.0221068	3.67	0.000
income	.0047535	.0007987	5.95	0.000
female	.5957393	.0444707	13.40	0.000
married	-.025713	.0463001	-0.56	0.579
physlim	.6045645	.057303	10.55	0.000
private	.868657	.0595024	14.60	0.000
chronic	1.022388	.0460539	22.20	0.000
_cons	-.8222869	.1047564	-7.85	0.000
/lnalpha	.502034	.0292317		
alpha	1.652078	.048293		

```
LR test of alpha=0: chibar2(01) = 1.7e+04              Prob >= chibar2 = 0.000
```

Type equation here.

Table 4.5 zero inflated poisson

```
. zip docvis $xlist,inf($xlist)nolog

Zero-inflated Poisson regression                Number of obs     =      4,412
                                                Nonzero obs       =      2,806
                                                Zero obs          =      1,606

Inflation model = logit                         LR chi2(7)        =    3208.89
Log likelihood  = -15723.76                     Prob > chi2       =     0.0000
```

docvis	Coef.	Std. Err.	z	P>|z|
docvis				
age	.0102207	.0077588	1.32	0.188
income	.0023525	.0002531	9.30	0.000
female	.2869789	.0164046	17.49	0.000
married	-.0825733	.0162368	-5.09	0.000
physlim	.3834317	.0173024	22.16	0.000
private	.3779918	.0287769	13.14	0.000
chronic	.6155628	.016581	37.12	0.000
_cons	.7978058	.0438202	18.21	0.000
inflate				
age	-.0914879	.0383143	-2.39	0.017
income	-.008962	.0014949	-5.99	0.000
female	-.9323071	.0764993	-12.19	0.000
married	-.2998978	.0781021	-3.84	0.000
physlim	-.5520193	.1153383	-4.79	0.000
private	-1.169697	.0931796	-12.55	0.000
chronic	-1.656834	.0962338	-17.22	0.000
_cons	2.056909	.170075	12.09	0.000

4.6 Zero Inflated Negative Binomial

The top half of the table contains coefficient for the factor change in the expected count for those in the not zero group. The bottom half, contains logit coefficients for the factor change in the odds of being in the always zero group /lnalpha is the natural log of alpha the dispersion parameter, alpha is the dispersion parameter of the count model.

4.7 Comparison Of The Models

The first table in the output summarizes the parameter estimates from each of the tested models, for each models we see the exponentiated coefficients and their t-statistics in the first block of the table. Then for each NBM we see the estimated dispersion parameters. Next for the zero-inflated models we see the estimates from the logistic model predicting the certain zeros. In the last block of the table, a set of fit-statistics is provided for each of the models. This includes the log-likelihood, BIC and AIC. Next we see a table with one line per model showing the maximum and mean differences in the observed versus predicted count. Next we have one table each of the models containing count by count information. In the last table, the tested models are compared to each other head to head using the tests appropriate to each comparison.

Fig. 4.1

The graph plots the residual from the tested models, the graph is used to eliminate a model that does not fit well.

Table 4.6 zero inflated negative binomial

```
zinb docvis $xlist,inf($xlist)nolog

zero-inflated negative binomial regression      Number of obs     =      4,412
                                                Nonzero obs       =      2,806
                                                Zero obs          =      1,606

Inflation model = logit                         LR chi2(7)        =     629.01
Log likelihood  = -9678.842                     Prob > chi2       =     0.0000
```

docvis	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
docvis						
age	.0410393	.0227682	1.80	0.071	-.0035856	.0856642
income	.0031686	.0007801	4.06	0.000	.0016397	.0046975
female	.3992868	.0472314	8.45	0.000	.3067149	.4918587
married	-.0755182	.0486933	-1.55	0.121	-.1709553	.0199189
physlim	.495781	.0554374	8.94	0.000	.3871257	.6044363
private	.507167	.0752578	6.74	0.000	.3596645	.6546694
chronic	.8193822	.0475597	17.23	0.000	.7261669	.9125975
_cons	.1009222	.1245542	0.81	0.418	-.1431996	.345044
inflate						
age	-.1537905	.096904	-1.59	0.113	-.3437188	.0361378
income	-.0177433	.0058397	-3.04	0.002	-.0291889	-.0062977
female	-1.58483	.22384	-7.08	0.000	-2.023548	-1.146111
married	-.551142	.1857786	-2.97	0.003	-.9152613	-.1870227
physlim	-.9756753	.3929877	-2.48	0.013	-1.745917	-.2054335
private	-1.650272	.2097979	-7.87	0.000	-2.061468	-1.239076
chronic	-3.173707	.7859099	-4.04	0.000	-4.714062	-1.633352
_cons	2.083506	.3923714	5.31	0.000	1.314472	2.85254
/lnalpha	.2777115	.0362869	7.65	0.000	.2065905	.3488324
alpha	1.320105	.0479025			1.229479	1.417412

able 4.7 comparison of the model

Variable	PRM	NBRM	ZIP	ZINB
docvis				
Age in years / 10	1.032	1.085	1.010	1.042
	4.06	3.67	1.32	1.80
Income in $ / 1000	1.004	1.005	1.002	1.003
	16.60	5.95	9.30	4.06
= 1 if female				
1	1.635	1.814	1.332	1.491
	30.50	13.40	17.49	8.45
= 1 if married				
1	0.982	0.975	0.921	0.927
	-1.13	-0.56	-5.09	-1.55
= 1 if physical limitation				
1	1.590	1.830	1.467	1.642
	26.66	10.55	22.16	8.94
= 1 if private insurance				
1	2.170	2.384	1.459	1.661
	27.81	14.60	13.14	6.74
= 1 if a chronic condition				
1	2.664	2.780	1.851	2.269
	59.41	22.20	37.12	17.23
Constant	0.668	0.439	2.221	1.106
	-9.79	-7.85	18.21	0.81
lnalpha				
Constant		1.652		1.320
		17.17		7.65
inflate				
Age in years / 10			0.913	0.857
			-2.39	-1.59
Income in $ / 1000			0.991	0.982
			-5.99	-3.04
= 1 if female				
1			0.394	0.205
			-12.19	-7.08
= 1 if married				
1			0.741	0.576
			-3.84	-2.97
= 1 if physical limitation				
1			0.576	0.377
			-4.79	-2.48
= 1 if private insurance				
1			0.310	0.192
			-12.55	-7.87
= 1 if a chronic condition				
1			0.191	0.042
			-17.22	-4.04
Constant			7.822	8.033
			12.09	5.31
Statistics				
alpha		1.652		
N	4412	4412	4412	4412
ll	-1.81e+04	-9782.220	-1.57e+04	-9678.842
bic	36305.795	19639.968	31581.800	19500.350
aic	36254.658	19582.439	31479.526	19391.685

Comparison of Mean Observed and Predicted Count

Model	Maximum Difference	At Value	Mean |Diff|
PRM	0.250	0	0.051
NBRM	0.008	5	0.003
ZIP	0.128	1	0.035
ZINB	0.028	1	0.008

PRM: Predicted and actual probabilities

Count	Actual	Predicted	|Diff|	Pearson
0	0.364	0.114	0.250	2426.826
1	0.159	0.173	0.014	4.989
2	0.104	0.166	0.062	102.237
3	0.068	0.132	0.064	138.096
4	0.054	0.097	0.043	84.451
5	0.046	0.070	0.024	35.967
6	0.029	0.052	0.024	47.901
7	0.029	0.041	0.012	14.440
8	0.021	0.033	0.012	18.881
9	0.017	0.027	0.010	16.002
Sum	0.891	0.906	0.515	2889.791

NBRM: Predicted and actual probabilities

ZINB: Predicted and actual probabilities

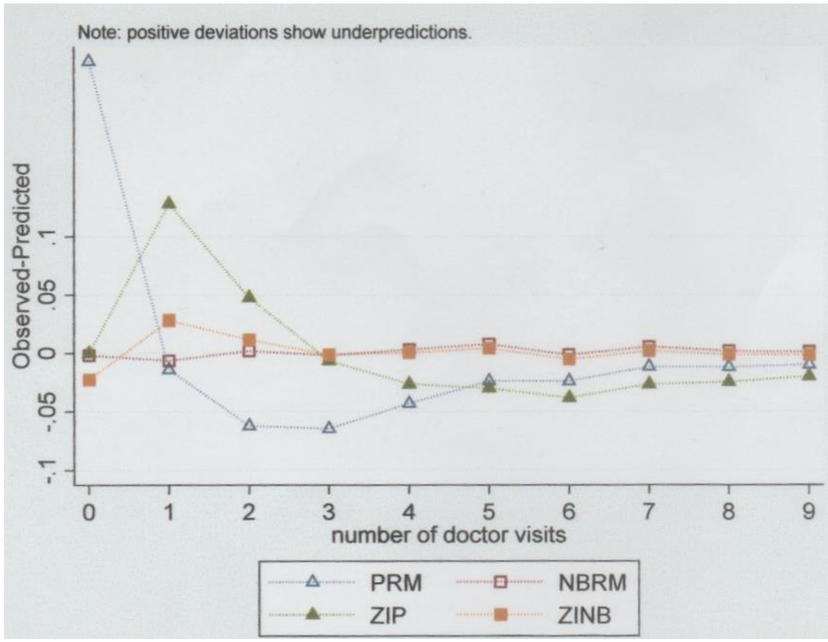
Count	Actual	Predicted	|Diff|	Pearson
0	0.364	0.387	0.023	5.797
1	0.159	0.130	0.028	27.141
2	0.104	0.093	0.011	6.221
3	0.068	0.069	0.001	0.129
4	0.054	0.054	0.000	0.009
5	0.046	0.042	0.004	1.637
6	0.029	0.034	0.005	3.670
7	0.029	0.027	0.002	0.630
8	0.021	0.023	0.001	0.305
9	0.017	0.019	0.001	0.506
Sum	0.891	0.877	0.078	46.045

Tests and Fit Statistics

PRM	BIC= 36305.795	AIC= 36254.658	Prefer	Over	Evidence
vs **NBRM**	BIC= 19639.968	dif= 16665.827	NBRM	PRM	Very strong
	AIC= 19582.439	dif= 16672.219	NBRM	PRM	
	LRX2=16674.219	prob= 0.000	NBRM	PRM	p=0.000
vs **ZIP**	BIC= 31581.800	dif= 4723.995	ZIP	PRM	Very strong
	AIC= 31479.526	dif= 4775.132	ZIP	PRM	
	Vuong= .	prob= .	ZIP	PRM	p=.
vs **ZINB**	BIC= 19500.350	dif= 16805.445	ZINB	PRM	Very strong
	AIC= 19391.685	dif= 16862.974	ZINB	PRM	
NBRM	BIC= 19639.968	AIC= 19582.439	Prefer	Over	Evidence
vs **ZIP**	BIC= 31581.800	dif=-11941.832	NBRM	ZIP	Very strong
	AIC= 31479.526	dif=-11897.087	NBRM	ZIP	
vs **ZINB**	BIC= 19500.350	dif= 139.618	ZINB	NBRM	Very strong
	AIC= 19391.685	dif= 190.755	ZINB	NBRM	
	Vuong= .	prob= .	ZINB	NBRM	p=.

ZIP	BIC= 31581.800	AIC= 31479.526	Prefer	Over	Evidence
vs **ZINB**	BIC= 19500.350	dif= 12081.450	ZINB	ZIP	Very strong
	AIC= 19391.685	dif= 12087.842	ZINB	ZIP	
	LRX2=12089.842	prob= 0.000	ZINB	ZIP	p=0.000

Fig. 4.1: A Graph that connect Observed-Predicted with Number of Doctor visits



DISCUSSION OF RESULTS

In line with our stated aim and objectives, this research work has been able to achieve the following;

i) It used data obtained from http://www.stata-press.com to model docvis among age, income, female, black, Hispanic, married, physlim, private, chronic.
ii) It has analyzed the data as fitted by poisson regression model(PRM), negative binomial regression model(NBRM), zero-inflated poisson(ZIP), zero-inflated negative binomial.
iii) Finally it has compared between the four model to discover which model fits perfectly.

The interpretation of table 4.1 gives a clear description of the data. Table 4.2 is the overall summary of the count data, the mean number of docvis is approximately 3.957 and the variance is $(3.957)^2 = 15.06$, which is substantially more than the mean this leads to our first regression. Table 4.3 Coef. - the poisson coefficient can be interpreted as follows; for a one unit change in the predictor variable, the difference in logs of the expected counts is expected to change by the respective regression coefficient, given the other predictor variables in the model are held constant. Age- this is the poisson regression estimate for a one unit increase in age, given the other variables are held constant in the model. If a patient were to increase docvis by one unit point, the difference in the logs of expected counts would be expected to increase by 0.032 unit, while holding the other variables constant. Income - this is the poisson regression estimate for one unit increase in income, given the other variables are held constant in the model. If a patient were to increase docvis by one unit point, the difference in the logs of expected counts would be expected to increase by 0.004

unit, while holding the other variables constant. Female - this is the poisson regression estimate for a one unit increase in number of female, given the other variables are held constant in the model. If a female patient were to increase docvis by one unit point, the difference in the logs of expected counts would be expected to increase by 0.49 unit, while holding the other variables constant. Married - this is the poisson regression estimate for a one unit increase in number of married patient, given the other variables are held constant in the model. If a married patient were to increase docvis by one unit point, the difference in the logs of expected counts would be expected to decrease by 0.02 unit, while holding the other variables constant. Physlim this is the poisson regression estimate for one unit increase in number of physlim patient, given the other variables are held constant in the model. If a physlim patient were to increase docvis by one unit point, the difference in the logs of expected counts would be expected to increase by 0.46 unit, while holding the other variables constant. From our output we see that private patient and chronic patients also increase their unit number of docvis by 0.77 and 0.98 respectively. _const. – this is poisson regression estimate when all variables are estimated at point zero. For age, income, female, married, physlim, private and chronic estimated at point zero. The log of the expected count of docvis is -0.40 unit. The standard error- these are the standard error of each of the poisson regression coefficient used to estimated the z test statistics.

The z-test statistics testing the slope for age on docvis is zero given the other variables are in the model, is (0.031/0.008) -4.06 with an associated P-value of < 0.0001 with alpha been set as 0.05, we would reject the null hypothesis and conclude that poisson regression coefficient for age is statistically different from zero. We see that income, female, physlim, private and chronic has an associated P-values <0.0001 with alpha been set as 0.05 we would reject the null hypothesis and conclude the their poisson regression coefficients is statistically different from zero. Except married which has an associated P-value of 0.260 with alpha set at 0.05, we would fail to reject the null hypothesis and conclude that the poisson regression coefficient for married is not statistically not different from zero. The estat gof which compares the observed distribution predicted by poisson distribution. The highly significant test statistics indicates that this is not a very good choice. Which leads to us running another analysis but this time we using negative binomial regression, as displayed in the next table. Table 4.4, the Coef.,Std. Err., z and P >|z| has same interpretation with the poisson regression estimate /lnalpha is the estimate of the log of dispersion parameter, alpha the estimate of the dispersion parameter. $\alpha = \exp(lnalpha)\ 1.652078.$ the dispersion parameter of alpha is greater than zero that is the data are over dispersed. The likelihood ratio test of alpha=0 at the bottom is the test of overdispersion parameter alpha. When the overdispersion parameter is equal to zero the test statistics is -2[-18119.329-(-9782.2197) = 16674.22 with an associated p-value 0f <0.0001. the large test statistic would suggest that the response variable is overdispersed and is not sufficiently described by the simpler poisson distribution. Table 4.5 From our result in the non-zero group we see that the z-test for predictors income, female, married, physlim, private, chronic and the intercept _cons are all statistically significant we would reject the null hypothesis and conclude that the regression coefficients has been found to be statistically different from zero, except the predictor age. For the zero group the z-test for the predictors age, income, female, married, physlim, private, chronic and the intercept _cons are all statistically significant so we would reject the null hypothesis and conclude that the regression coefficients has been found to be statistically different from zero. Table 4.6 from our result In the non-zero group the predicted number of docvis among income, female, physlim, private, chronic are significant except married. The bottom half, contains logit coefficients for the factor change in the odds of being in the always zero group. The predicted number of docvis among the variables is statistically significant except age. alpha the dispersion parameter is greater than zero, this suggests that our data is overdispersed. Table 4.7 from the last block of the model parameter and fit we see that the two model aic and bic are extremely close.the parameter estimates are nearly identical. from the residuals by count table we see that the NBRM and ZINB did better at this predicted and overall the two models had a lower mean absolute differences between the predicted and observed values. At this point the NBM and ZINB is looking more appropriate than the PRM and ZIP. Next we have one table for each of the models containing counts by count information. In these four tables, shows the output for counts 0-9, the actual proportion from each models, the absolute differences is included, as in the given counts distribution to a pearson chi-sqaure statistics comparing that actual distribution of the data and the distribution proposed by the model for a given row. The pearson statistics which is calculate as $N(|diff|)^{2/}/predicted$, where N is the number of observations in the dataset. Looking closely at the sum of the pearson columns gives us a sense of how close the predicted proportions were to the actual proportions using this method to compare, the NBRM and ZINB appears better than the PRM and ZIP. Finally in the next the result suggests which model is most preferred by the given comparison strength of the evidences supporting this preference. When we compare the four models using BIC and AIC the NBRM and ZINB is preferred over PRM and ZIP. Fig 4.1 is a graph that plots the residuals from the tested models, the models with lines closest to zero should be considered for our data, at the zero and one count NBRM and ZINB appears better than the PRM and ZIP models.

6.1 CONCLUSION

Categorically, this work can be summarized without any fear of contradiction the poisson regression model (PRM), negative binomial regression model(NBRM) which is the base for the other regression model zero-inflated poisson (ZIP) and zero-inflated(ZINB) respectively fits this work except the fact that some models fits better than the others. The PRM doesn't fit reasonably well because if its strict conditions of equal conditional mean and equal conditional variances E(x) = Var(x). as a result leading to under predictions of zeros. While NBRM because of its flexibility fits reasonable well because it allows the variances to be greater than the mean called over-dispersion. The zero-inflated model assumes two groups, one has no chance of going beyond zeros. The other group may have zero count but the probability of having a positive count is non-zero. In conclusion the ZINB and NBRM fits better than the other two models.

6.2 RECOMMENDATION

Recommendations are hereby presented;

1) NBRM often maybe good enough for the modeling of count data so the need of zero inflated models might be questioned.
2) Because of PRM strict conditions it makes its result to be inconsistent and biased.
3) This work can furthered for verification purposes or contribution by any researcher who picks interest in it.

6.3 CONTRIBUTION TO KNOWLEDGE

The use of Poisson Regression Model with application to count data ease the analysis, but because poisson regression model cannot handle overdispersion, its results are inconsistent and baised. It is seen that negative binomial model because of its flexibility its results are more consistent which maybe suitable for modeling count data. In the case of several zeros, zero inflated negative binomial model is preferred to pure negative binomial model.

REFERENCES

Agresti, A. (2002*). Categorical data analysis. 2nd edition*. New York, Wiley.

Aiken, L. S. and West, S. G. (1991). *Multiple Regression: testing and interpreting interactions*, Newbury Park, CA: Sage.

Alexander, N., R. Moyeed *and* J. Stander. *(*2000*).* Spatial modeling of individual-level parasite counts using the negative binomial distribution. *Biostatistics 1:* 453–*463.*

Bailer A.J. and L.T. Stayner (1997). Modeling fatal injury rates using poisson regression. *Journal of safety research;28:177-186.*

Bair, H. (2013). Poisson regression: Lack of fit ≠ Overdispersion, StatNews #86, cornell University, http://www.cscu.cornell.edu/news/ststnews/stnews86.pdf .

Beckett,S., Jee, J., Ncube, T.,Pompilus, S., Washington, Q., Singh, A., Pal, N. (2014). Zero-inflated poisson (ZIP) distribution: parameter estimated and applications to model data from natural calamities. Involve*: A Journal of Mathematic*s 7(6):751-767.

Cameron, A. C. and Trivedi, P. K. (1998). *Regression Analysis of Count Data*. NewYork: Cambridge Press.

Cameron, A. C. and Trivedi, P. K. (2009). *Microeconometrics Using Stata*. CollegeStation, TX: Stata Press.

Cameron, A. C. (2008). *Advances in Count Data Regression Talk for the Applied Statistics Workshop,March* 28, 2009. http://cameron.econ.ucdavis.edu/racd/count.html.

Cameron, A.C. and Trivedi, P.K. (1986). Econometric models on count data comparison and applications of some estimates and tests. *Journal of Applied* Econometrics, 1(1), 29-53.

Dobson,A.J. (2002). *An introduction to generalized linear model,2nd ed.*, New York: Chapman & Hall/CRC.

Dupont, W. D. (2002). Statistical Modeling for Biomedical Researchers*: A Simple Introduction to the Analysis of Complex Data.* New York: Cambridge Press.

Famoye, F., Wulu, J. T. Jr.,and K. P. Singh. (2004). On the generalized Poisson regression model with an application to accident data. *Journal of Data Science 2:* 287*–295.*

Ferenc M., Rita Hegedus (2014). The use of poisson regression in the sociological study of suicide. Journal of sociology and social policy;Vol.5,No2

Gardner, W., Mulvey, E.P. and Shaw E. C. (1995). Regression analyses of counts and rates: poisson, overdispersed poisson and negative binomial models. *Phychological Bulletin*;*118: 392-404.*

Greene, W. H. (1994). *Econometrics analysis*. New York: Stern School of Business, New York University, Department of Economics.

Hall, D. B. and Zhengang, Z. (2004). Marginal models for zero inflated clustered data. *Statistical modeling, 4:161-180.*

Hilbe, J.M. (2011). *Negative binomial regression*. Cambridge University Press, Cambridge

Joseph M. Hilbe (2014). *Modeling count data*. Cambridge University press, Cambridge.

Jerald F. Lawless (1987). Negative binomial and mixed poisson regression. *Journal of statistics; 15: https://doi.org/10.2307/3314912*

Long, J. S. (1997). *Regression Models for Categorical and Limited Dependent Variables.*Thousand Oaks, CA: Sage Publications.

Long, J. S. and Freese, J. (2006). *Regression Models for Categorical Dependent Variables Using Stata, Second Edition*. College Station, TX: Stata Press

Monday O. Adenomon (2017). Fitting a poisson regression model to reported deaths from HIV/AIDS in Nigeria. *Journal of statistical distributions and applications; 3(3):56-60.*

Mc Cullagh, P. and Nelder, J.A. (1989). *Generalized Linear Models, 2nd ed*. London; Chapman and Hall.

Sileshi, G., G. Hailu,*and* G. I. Nyadzi. *(2009).* Traditional occupancy–abundance models are inadequate for zero-inflated ecological count data. *Ecological Modeling 220:* 1764–*1775.*

White, G. C.,*and* R. E. Bennetts. 1996. Analysis of frequency count data using the negative binomial distribution. *Ecology 77:* 2549–*2557.*

Wan F. (2011). Applying fixed effects panel count model to examine road accident occurrence. Journal of applied-science;11:1185-1191.

Ver Hoef, J. and Boveng, P. L. (2007). *Quasi-poisson* vs *Negative Binomial Regression: How should we model overdispersed count data?* Ecology88:2766-2772. http://doi.org/10.1890/07-0043.1

Zamani, H., Ismail, N. (2013). Score test for testing zero-inflated poisson regression against zero-inflated generalized poisson alternatives. *Journal of Applied Statistics* 40(9):2056-2068